

Due date: Whatever UML says. Please remember I will not accept an assignment, *under any circumstances*, posted after I posted my solutions.

Try to explain first in English what you are doing, before jumping into formulas.

Your answers should be in ONE PDF file (that is ONE file, and it is not a JPG, HEIC or whatever other format). All pages in your file have to be in portrait orientation. Pages that are rotated 90 (or any other angle except zero) degrees will not be marked.

Max is a utility-maximizer consumer with a utility function $U(x, y) = x + \sqrt{y}$ and the usual budget constraint $M = p_x x + p_y y$, where M is his income, p_x and p_y is the price of good x and y , respectively.

1. Write down Max's optimization problem as an optimization problem with a single variable, y . (Hint: solve the budget constraint for x , and plug the solution for x into the objective function.)
2. Write down Max's first order condition(s) for an interior solution.
3. Use the Implicit Function Theorem with the first order condition to calculate all the partial derivatives of the optimal value of y . Note: you have to calculate these derivatives without calculating the optimal value of y !
4. Find the optimal value of y and use it to verify your results from question 3.
5. Write down the second order condition(s) and check whether it/they are satisfied.

Of course Max cannot consume a negative value of y , and he also cannot consume more than the maximum possible quantity of y , M/p_y .

6. Is it possible Max has a corner solution $y = 0$? How about a corner solution $y = M/p_y$? If yes, for which values of prices and income are they possible?

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7. Write down Max's optimization problem as a constrained optimization problem.
 8. Write down the first order conditions for Max's constrained optimization problem.
 9. Use the implicit functions procedure on the first order conditions to calculate the derivatives of Max's demand for good y with respect to M , p_x and p_y . Note: you have to calculate those partial derivatives without finding the demand for x or y !
 10. Write down the second order condition(s) and check whether it/they is/are satisfied.
 11. Find Max's demand for good x and y .
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